## Geometric phase and topology in a field gradient matter-wave interferometer

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## 1. Introduction

The geometric phase due to the evolution of the Hamiltonian is a central concept in quantum physics and may become advantageous for quantum technology. In noncyclic evolutions, a proposition relates the geometric phase to the area bounded by the phase-space trajectory and the shortest geodesic connecting its endpoints. We report: a previously unshown experimental confirmation of the geodesic rule for a non-cyclic geometric phase using a spatial $\mathrm{SU}(2)$ matterwave interferometer, demonstrating, with high precision, the predicted phase sign change and $п$ jumps [1]
Momentum states exhibit steps like Shapiro steps in Josephson junctions, driven by geometric phase and topology.
3. Non-cyclic geometric phase and geodesic rule


The geodesic rule - the evolution back along any geodesic curve joining B to $A$. The enclosed area corresponds to the geometric phase. [2,3]

4. $\Pi$ phase jump at the equator and phase rigidity within hemisphere
 (A) Population transfer to state 1 1)
versus the duration of the RF radiation pulse.
$\mathrm{NH}:$ Northern hemisphere. Scanning $\boldsymbol{\theta}$ $\mathrm{SH}:$ Southern hemisphere. $(20 \mu \mathrm{~S} \rightarrow \pi)$ $\pi$ phase jump \& phase rigidity (when $\Delta \boldsymbol{\phi}=\boldsymbol{\pi}$ )
(B) The interference phase as a (B) The interference of Rabi oscillation.

(c) Averaged CCD image of interference when the Bloch vectors are all in the northern hemisphere (D) Averaged picture of the second half of
the data, in which the Bloch vectors are all pointing in the southern hemisphere. (E) Averaged picture of all the data.

5. Geometric phase jump - "phase slip"

(a), the measured total phase when $\mathrm{TR}_{3}=\Pi / 2$. Here the relative rotation between the two vectors is decided by $\Delta \varphi=\Delta \mathrm{E} \times \mathrm{TG}_{3} / \hbar$. As the two-level system consists of the $\mathrm{mF}=1$ and $\mathrm{mF}=2$ states, there is an additional phase resulting from $\mathrm{TG}_{3}$. The phase jump around $\mathrm{TG}_{3}=10 \mu \mathrm{~s}$ is obvious. (b), the geometric phase with the strip of the dynamical phase. The phase rigidity and the $n$ phase jump are clearly shown; (c), the corresponding change of the visibility, as $|\operatorname{Cos}(\Delta \varphi) / 2|$, which also implies the boundary conditions. (d) geodesic connection (purple).

7. Momentum steps and Shapiro steps
 $[4,5]$



## 8. References

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